Student Number:

Sydney Girls High School
2022
TRIAL HIGHER SCHOOL CERTIFICATE

EXAMINATION

Mathematics

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

Total Marks:

100

Section I – 10 marks (pages 4–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 10–43)

- Attempt Questions 11–34
- Allow about 2 hours and 45 minutes for this section

THIS IS A TRIAL PAPER ONLY
It does not necessarily reflect the format or the content of the 2022 HSC Examination Paper in this subject.

Question	**1-10 M.C**	**11-16**	**17-21**	**22-26**	**27-34**	
Total	**/10**	**/16**	**/19**	**/19**	**/36**	**%**

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

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1 The interval which gives the range of the function $y = 3\sin 2x + 4$ is:

A. $[3, 7]$

B. $[4, 6]$

C. $[1, 7]$

D. $[4, 9]$

2 The best description of $x = y^2 + 1$ is:

A. one to one

B. many to one

C. one to many

D. many to many

3 The first three terms of an arithmetic series are $2, 7$ and 12. The 15^{th} term of the series is:

A. 72

B. 77

C. 555

D. 595

4 The anti-derivative of $2^x \ln 4$ is:

A. $\frac{2^x}{2\ln 2} + c$

B. $2^{2x} + c$

C. $2^{x+1} + c$

D. $\frac{2^x}{3\ln 2} + c$

5 On the Richter scale, the magnitude R of an earthquake of intensity I is given by the formula $R = \log_{10}\left(\frac{I}{I_0}\right)$, where I_0 is a reference intensity used for comparisons.
The 1989 Newcastle Earthquake measured a magnitude of 5.6 on the Richter scale.
Which of these is closest to $\left(\frac{I}{I_0}\right)$, which is the number of times this earthquake is more intense than the reference intensity?

A. 400 times

B. 4000 times

C. 40000 times

D. 400000 times

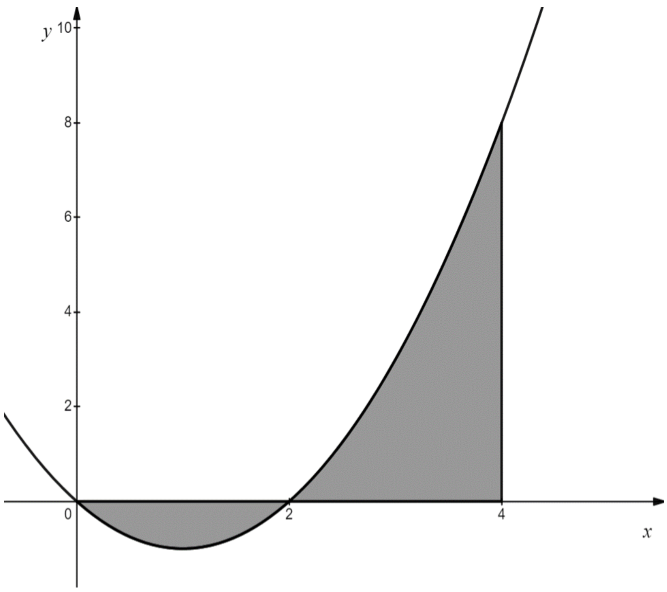
6 The graph of the parabola $y = x^2 - 2x$ is given. The shaded area is equal to:

A. $-1\frac{1}{3}$ units^2

B. $5\frac{1}{3}$ units^2

C. 8 units^2

D. $6\frac{2}{3}$ units^2



7 How many terms are there in the following arithmetic sequence:

$$-12, -10\frac{1}{2}, -9, \dots, 108$$

A. 67

B. 73

C. 79

D. 81

8 A and B are events of a sample space.

Given that $P(B|A) = \sqrt{p}$, $P(A) = p$ and $P(B) = p^2$, which of these is an expression for $P(A|B)$?

A. $p^{\frac{3}{2}}$

B. $\sqrt{p}$

C. $\frac{1}{p}$

D. $\frac{1}{\sqrt{p}}$

9 A continuous random variable, X, has the following probability density function, where $k > 0$.

$$f(x) = \begin{cases} \sin\left(\frac{\pi}{2} - 2x\right) & \text{for } -k \le x \le k \\ 0 & \text{for all other values} \end{cases}$$

Which of the following is true?

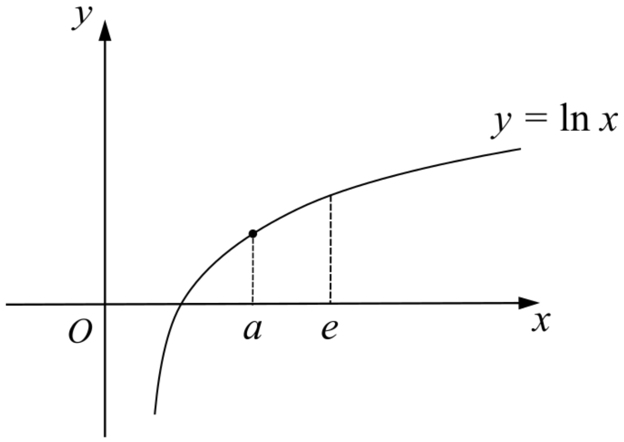
A. $P(X > 0.2) = 1 - P(X \le -0.2)$

B. $P(X > 0.2) = P(X < -0.2)$

C. $P(X > 0.2) = P(-0.2 < X < 0.2)$

D. $P(X > 0.2) = P(0 < X < 0.2)$

10 The line $y = mx + c$ is a tangent to the curve $y = lnx$ at the point where $x = a$, as shown in the diagram.



Which of the following statements is true?

A. $\frac{1}{e} < m < 1$ and $-1 < c < 0$

B. $1 < m < e$ and $-1 < c < 0$

C. $\frac{1}{e} < m < 1$ and $0 < c < 1$

D. $1 < m < e$ and $0 < c < 1$

Student Number:									

Mathematics Advanced

Section II 90 marks

Attempt Questions 11–34

Allow about 2 hours and 45 minutes for this section

Section II Answer Booklet 1

Attempt Questions 11 – 26 (54 marks)

Instructions:

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces do not provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the question booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

Find integers a and b such that $(3-\sqrt{2})^2 = a + b\sqrt{2}$. 2

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Question 12 (2 marks)

Solve the equation $|1-2x| = 5$. 2

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Question 13 (4 marks)

a) Find $\int \frac{x^2}{4x^3 - 6}\,dx$ 2

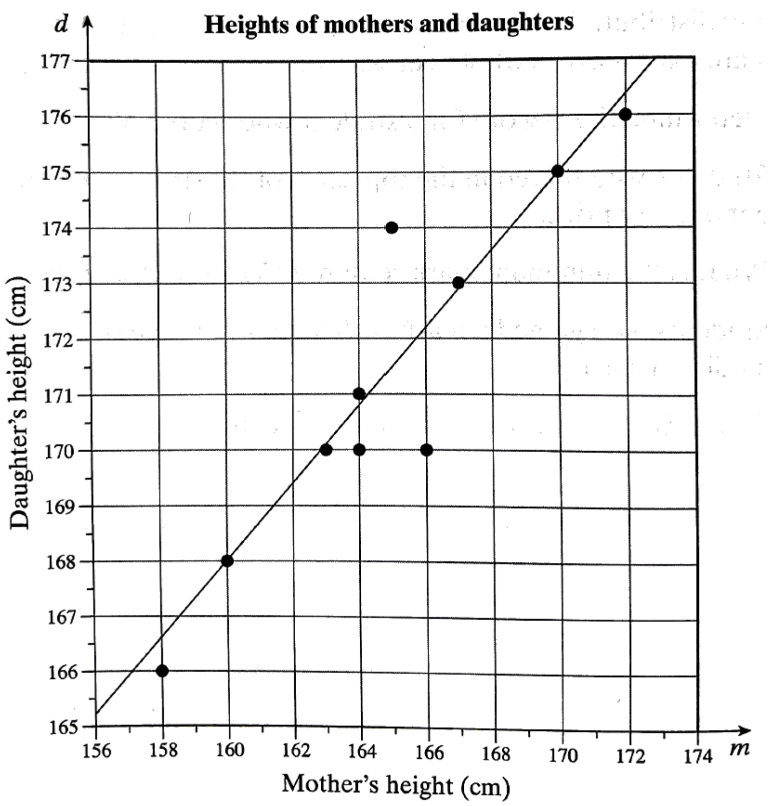
b) Find $\frac{d}{dx}\left(\frac{\tan x}{e^{2x}}\right)$. Answer in the simplest exact form. 2

Question 14 (3 marks)

The heights (cm) of 10 mothers (m) and their daughters (d) were recorded in the table below.

Mother's height in cm (m)	170	163	160	172	164	158	164	166	167	165
Daughter's height in cm (d)	175	170	168	176	170	166	171	170	173	174

The data was used to create a scatterplot and Amelia constructed a regression line by eye, as shown below:



a) Determine the equation of Amelia's regression line. 1

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b) Find the value of r, the Pearson's correlation coefficient (correct to 2 decimal places) and hence describe the relationship. 2

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Question 15 (3 marks)

Evaluate $\int_{0}^{\frac{\pi}{6}}\left(2\sin x-\sec^{2}2x\right)dx$. Leave your answer in the simplest exact form. 3

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Question 16 (2 marks)

Find the value of $\lim_{x\to 4}\dfrac{x^{2}-16}{8+2x-x^{2}}$ 2

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Question 17 (6 marks)

Chandra is an enthusiastic gardener. She planted a lemon tree on 1st January 2019, when it was 80 centimetres tall. At the end of the first year after planting, it was 130 centimetres tall, that is, it grew 50 centimetres. Each year's growth was then 90% of the growth of the previous year.

a) How tall was the lemon tree after three years? 2

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b) Assuming the present growth pattern is maintained, explain why the lemon tree will never reach a height of 10 metres? 2

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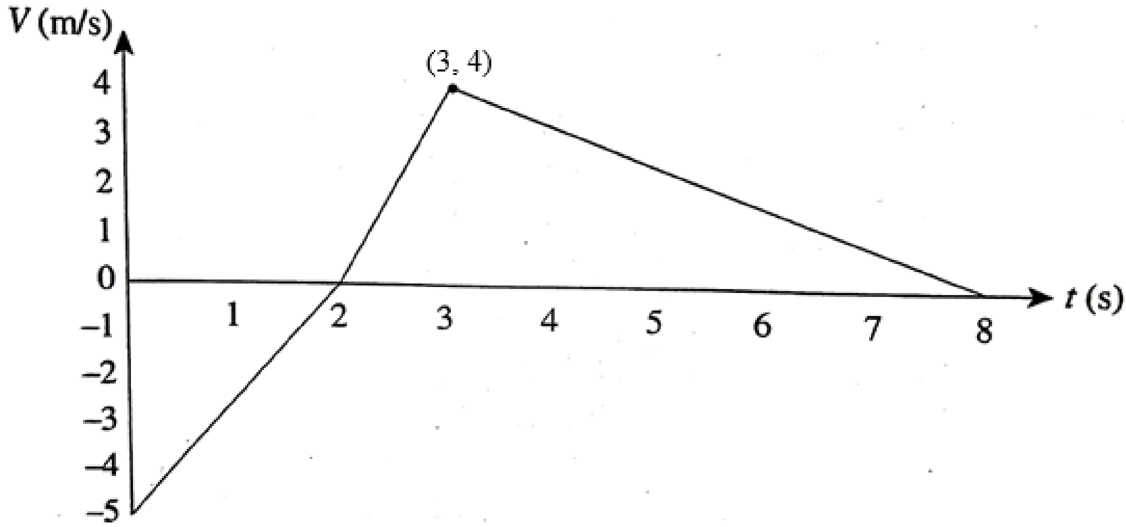
c) In which year will the lemon tree reach a height of 5 metres? 2

Question 18 (3 marks)

The graph below is composed of three line segments and shows the velocity V of a particle which is moving in a straight line. The velocity is given in metres per second at time t seconds, where $0 \leq t \leq 8$.



a) Determine the total distance covered by the particle during the 8 seconds. 2

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b) What is the particle's position relative to its starting position when $t = 8$ seconds? 1

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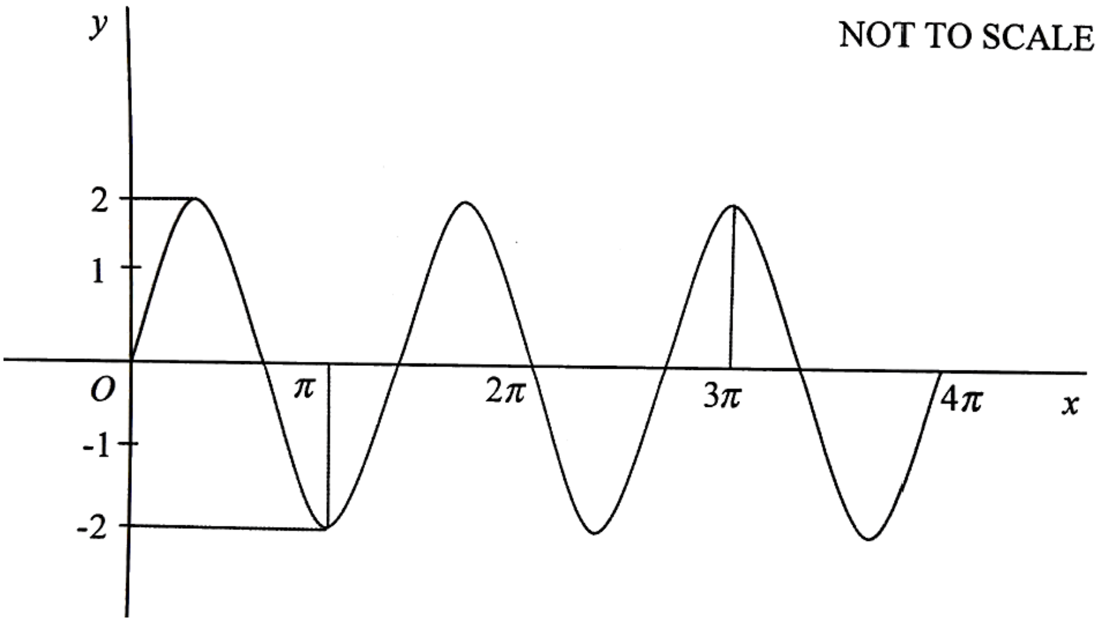
Question 19 (2 marks)

Find the equation for the graph given below: 2



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Question 20 (4 marks)

Packets of coffee beans are labelled with a net weight of 300 g. It is found that the weight of a packet can be modelled by a normal distribution, with mean 306 g and standard deviation 3 g.

a) Use the Empirical rule to determine the probability that the weight of one packet of coffee beans is less than the advertised weight of 300 g. 1

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b) In a shipment of 40 boxes, each with 100 packets of coffee beans, how many packets would be expected to be underweight? 1

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c) Manufacturers aim to ensure that the expected number of underweight packets in the shipment will be less than 20. The machine is adjusted to give a mean weight of 308 g, with the standard deviation of 3 g remaining the same. Will they meet their target? Justify your answer. 2

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Table : The standard normal distribution

The table below provides some values of the probabilities for the standard normal distribution.

$$\text{i.e.}\ \ \Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \phi(t)dt$$

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56360	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983

Question 21 (4 marks)

The probability it will rain on any day in June is 0.15. Two days in June are selected at random.

a) Complete the probability distribution table, where the variable X represents the number of rainy days across the 2 days: 1

x	0	1	2
$P(X = x)$			

b) Calculate, correct to two decimal places:

i) the expected value. 1

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ii) the variance and standard deviation. 2

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Question 22 (4 marks)

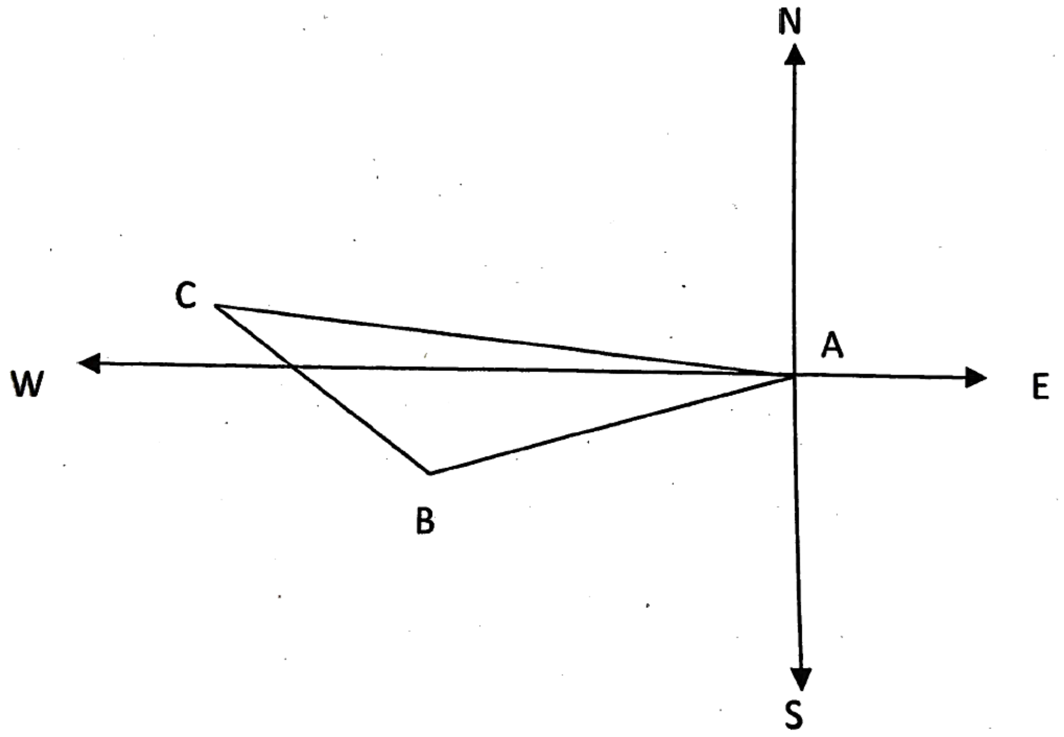
Solve the equation $\sec\theta + 8\cos\theta = 6$ for $0 \leq \theta \leq 2\pi$. Answer correct to 3 decimal places, where necessary. 4

Question 23 (4 marks)

A plane takes off from an airport (A) and travels in a direction of 258° for 3050 kilometres.

The plane lands at (B) and then heads in a direction of 300° for 2680 kilometres, landing at (C).

Use the diagram below to mark the given information.



a) Find the distance the plane must travel to return to A. Answer to the nearest kilometre. 2

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b) Find the bearing that the plane must travel from C to A. 2

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Question 24 (3 marks)

The table below shows the future values of an annuity of $1 for different rates of interest and for different numbers of compounding periods. The contributions are made at the end of each compounding period.

Future Value Interest Factors

Time Period	*Interest Rate*				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

An annuity account is opened and contributions of $500 are made at the end of every six months for 5 years.

For the first 4 years, the interest rate is 6% per annum, compounding six-monthly. For the 5th year, the interest rate increases to 8% per annum, compounding six-monthly.

Calculate the amount in the account immediately after the last contribution is made. 3

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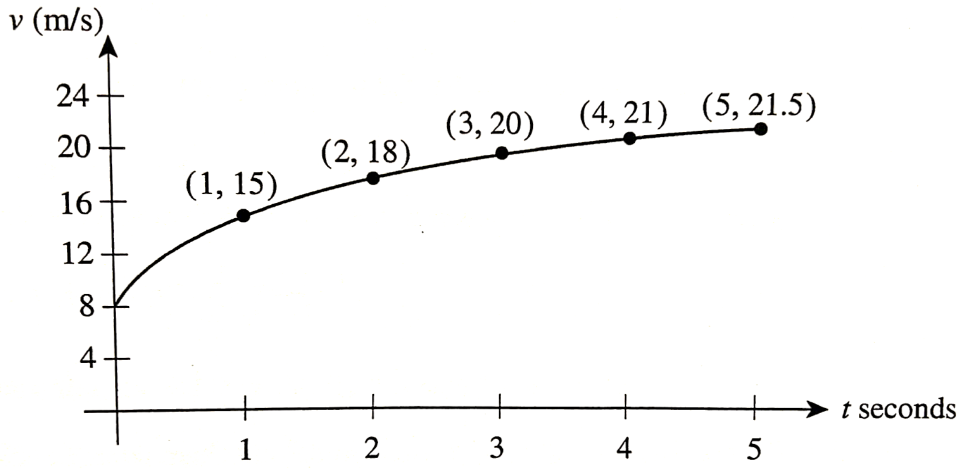
Question 25 (4 marks)

The diagram shows the graph of a particle's velocity, v m/s, at time t seconds.



a) Use the trapezoidal rule with 3 sub-intervals (4 function values) to approximate the distance the particle travels in the first 3 seconds. 2

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b) Is the estimate for your answer in part a) more than or less than the exact distance that the particle travels in the first 3 seconds? Justify your answer. 2

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Question 26 (4 marks)

Consider the cubic function $y = x^3 + ax^2 + bx + 3$, where a and b are integers.

At the point $(1, 8)$ on the curve, the equation of the tangent is given by $y = 2x + 6$.

Determine the values of a and b. 4

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student Number:									

Mathematics Advanced

Section II 90 marks

Attempt Questions 11–34

Allow about 2 hours and 45 minutes for this section

Section II Answer Booklet 2

Attempt Questions 27 – 34 (36 marks)

Instructions:

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces do not provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the question booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 27 (6 marks)

A probability density function is given by $f(x)=\begin{cases}\dfrac{a}{\sqrt{1+4x}}; & 0\le x\le 2\\ 0 & ; \text{otherwise}\end{cases}$ where a is a constant.

a) Show that $a = 1$. 2

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b) Find the cumulative distribution function, $F(x)$. 2

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c) Find the median. 2

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Question 28 (3 marks)

The rate of 'flu infection' in a population of a city is proportional to the number of infected individuals. That is the number of infected people F after t weeks is modelled by the equation $F = F_0 e^{kt}$ where F_0 and k are constants. It is known that after 3 weeks, there is twice the number of infections to begin with. If there were 1000 cases of flu infection originally, how many are there after 7 weeks? Express your answer correct to three significant figures. 3

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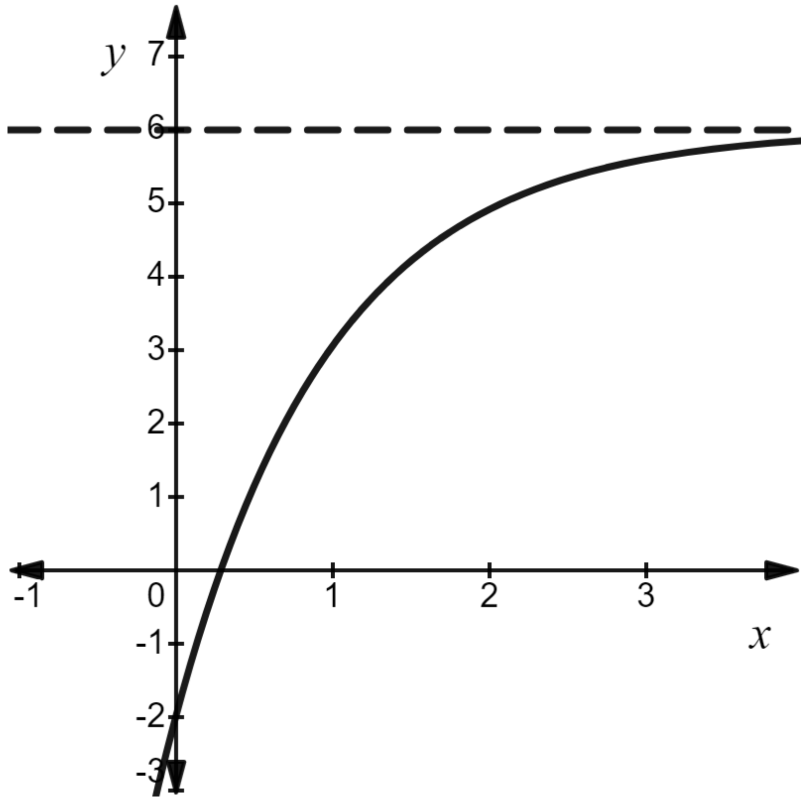
Question 29 (6 marks)

The diagram shows the graph of $y = 6 - 8e^{-x}$.



a) On the diagram, sketch the graph of $y = e^x$, which passes through the point $(1, e)$. Show that the x coordinates of the two points of intersection, (x_1 and x_2, where $x_1 < x_2$), are the solutions of the equation $e^{2x} - 6e^x + 8 = 0$ and solve this equation to find the exact values of x_1 and x_2. 4

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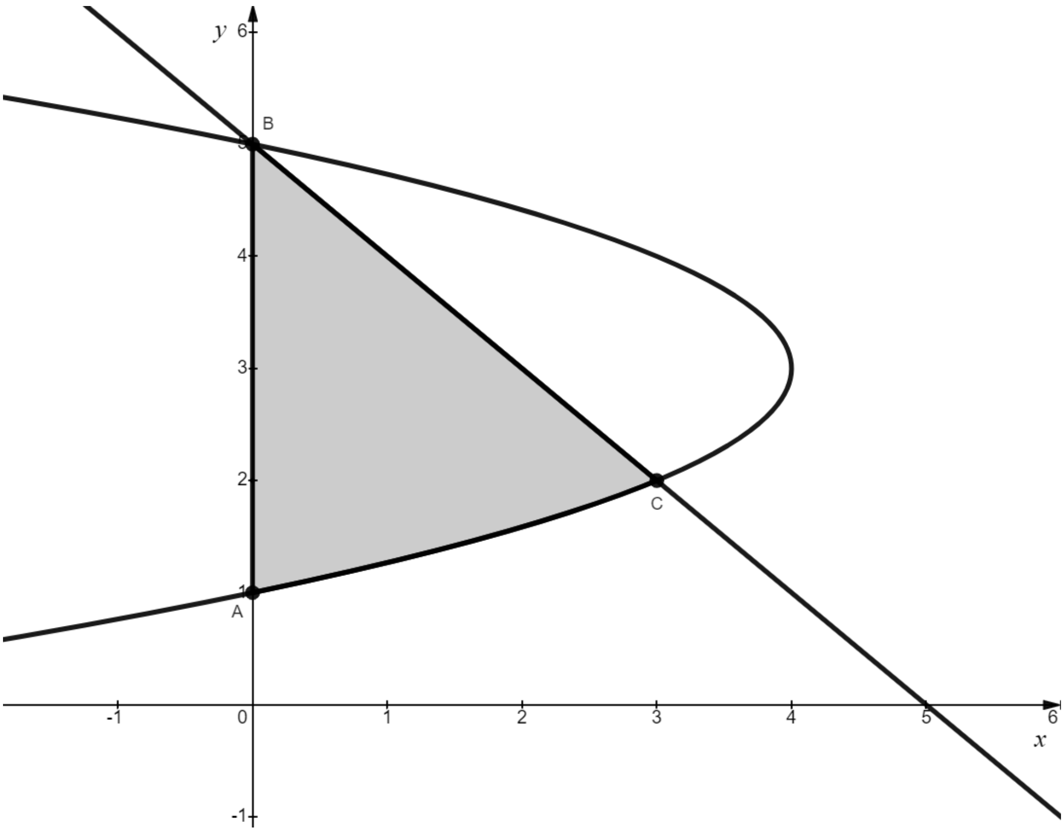
b) Find in simplest exact form, the area of the region enclosed by the two curves. 2

Question 30 (3 marks)

In the diagram below, the parabola $x=-y^2+6y-5$ meets the y −axis at points $A(0,1)$ and $B(0,5)$. The line $y=-x+5$ meets the parabola at points B and $C(3,2)$. Find the shaded area, which is bounded by the parabola, the line and the y −axis. Leave your answer as a simplified fraction. 3



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Question 31 (5 marks)

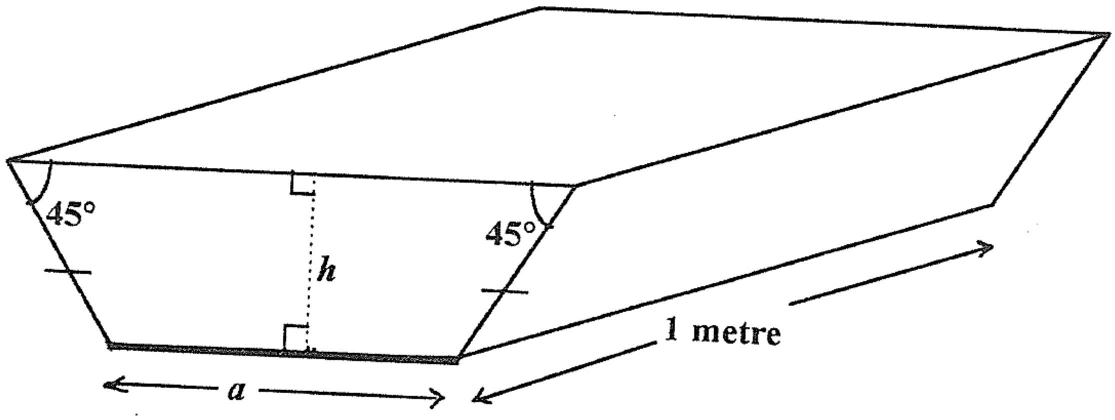
Karen is retiring next week and her Superannuation Fund contains $1 200 000. The Fund is earning 6% p.a. compound interest, compounding monthly. To cover her living expenses in her retirement, Karen wishes to withdraw a regular amount of $8 000 at the end of each month, after interest has been added.

a) Show that after 3 months the amount in her account A_3 is given by:

$$A_3 = 1\,200\,000(1.005)^3 - 8\,000[(1.005)^2 + (1.005) + 1]$$ 2

b) By finding a similar expression for the amount remaining after n months, find how many years the money will last. 3

Question 32 (6 marks)



(DIAGRAM NOT DRAWN TO SCALE)

An open trough of depth h metres and length one metre is constructed out of stainless steel sheeting.

The cross-section of the trough is an isosceles trapezium with the acute angles being 45° each.

The width of the bottom of the trough is a metres. The area of the cross-section measures 60 m^2.

a) Show that $a = \frac{60}{h} - h$. 2

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b) Show that the amount of stainless steel, A, in m^2, required to construct the trough is given by:

$$A = \frac{60}{h} - h + 2h\sqrt{2} + 120.$$ 2

c) Find the depth of the trough, to the nearest millimetre (mm), if the amount of stainless steel used is kept to a minimum. 2

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Question 33 (5 marks)

A prototype rocket which is initially at rest, takes off from a launchpad on the ground.

It has a time of flight of T seconds, and t is the time in seconds, where $0 \leq t \leq T$.

The velocity of the rocket, v ms^{-1}, is given by:

$$v(t) = 0.5e^{t}\sin\left(\frac{\pi t}{10}\right)$$

a) Shortly after the rocket takes off, the engine stops and it begins to descend towards the ground. Find the time at which the rocket begins to descend. 2

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b) Before the rocket starts to descend it reaches its maximum velocity. Find the time it takes for the rocket to achieve its maximum velocity. Give your answer correct to two decimal places. 3

Question 34 (2 marks)

The Artful Dodger has three 20 cent coins in his pocket. One of the coins has heads on both sides, another is biased such that it has a 75% chance of landing on heads and the third coin is a fair coin.

A coin is selected at random and tossed.

Given that the coin that was tossed comes up heads, what is the probability that it was the fair coin? 2

The End.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student Number:									
S	O	L	U	T	I	O	N	S	

Sydney Girls High School
2022
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION
Mathematics

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

Total Marks:

100

Section I – 10 marks (pages 4–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 10–43)

- Attempt Questions 11–34
- Allow about 2 hours and 45 minutes for this section

THIS IS A TRIAL PAPER ONLY
It does not necessarily reflect the format or the content of the 2022 HSC Examination Paper in this subject.

Question	1-10 M.C	11-16	17-21	22-26	27-34	
Total	/10	/16	/19	/19	/36	%

Sydney Girls High School

Mathematics Faculty

Multiple Choice Answer Sheet
Trial HSC Mathematics Advanced

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = ? (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ●✗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct ↙

A ●✗ B ●✗ C ○ D ○

Completely fill the response oval representing the most correct answer.

1. A ○ B ○ C ● D ○
2. A ○ B ○ C ● D ○
3. A ● B ○ C ○ D ○
4. A ○ B ○ C ● D ○
5. A ○ B ○ C ○ D ●
6. A ○ B ○ C ● D ○
7. A ○ B ○ C ○ D ●
8. A ○ B ○ C ○ D ●
9. A ○ B ● C ○ D ○
10. A ● B ○ C ○ D ○

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

---

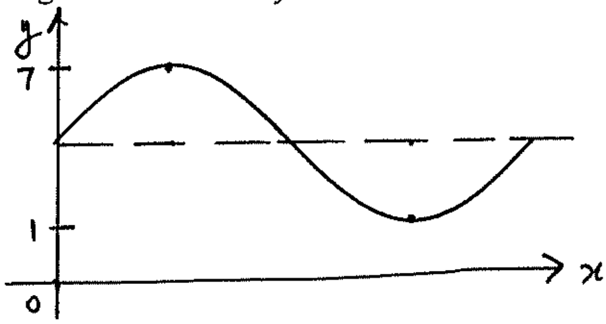
1 The interval which gives the range of the function $y = 3\sin 2x + 4$ is:

A. $[3, 7]$

B. $[4, 6]$

(C.) $[1, 7]$

D. $[4, 9]$



Not to scale.

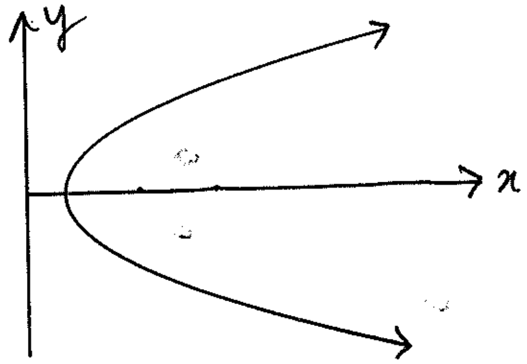
2 The best description of $x = y^2 + 1$ is:

A. one to one

B. many to one

(C.) one to many

D. many to many



3 The first three terms of an arithmetic series are 2, 7 and 12. The 15[th] term of the series is:

(A.) 72

B. 77

C. 555

D. 595

$a = 2$
$d = 5$
$n = 15$

$T_n = a + (n-1)d$
$T_{15} = 2 + (15-1) \times 5$
$T_{15} = 72.$

4 The anti-derivative of $2^x \ln 4$ is:

A. $\frac{2^x}{2\ln 2} + c$

B. $2^{2x} + c$

(C.) $2^{x+1} + c$

D. $\frac{2^x}{3\ln 2} + c$

$\int 2^x \ln 4 \, dx$
$= \ln 4 \int 2^x \, dx$
$= \ln 2^2 \times \frac{2^x}{\ln 2} + c$
$= 2\ln 2 \times \frac{2^x}{\ln 2} + c$
$= 2 \times 2^x + c$
$= 2^{x+1} + c$

5 On the Richter scale, the magnitude R of an earthquake of intensity I is given by the formula $R = \log_{10}\left(\frac{I}{I_0}\right)$, where I_0 is a reference intensity used for comparisons.
The 1989 Newcastle Earthquake measured a magnitude of 5.6 on the Richter scale.
Which of these is closest to $\left(\frac{I}{I_0}\right)$, which is the number of times this earthquake is more intense than the reference intensity?

A. 400 times

B. 4000 times

C. 40000 times

(D.) 400000 times

$5.6 = \log_{10}\left(\frac{I}{I_0}\right)$
$\frac{I}{I_0} = 10^{5.6}$
$= 398107.1706$
$\doteqdot$ 400 000 times (nearest hundred thousand).

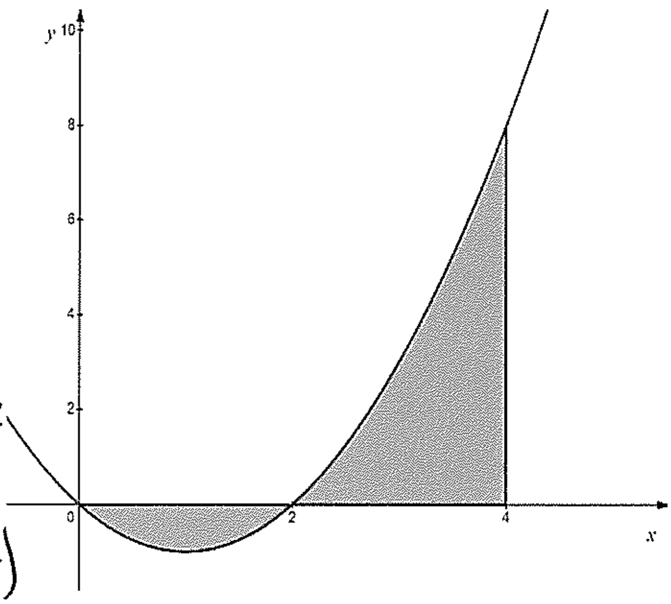
6 The graph of the parabola $y = x^2 - 2x$ is given. The shaded area is equal to:

A. $-1\frac{1}{3}$ units^2

B. $5\frac{1}{3}$ units^2

(C.) 8 units^2

D. $6\frac{2}{3}$ units^2



$\left|\int_0^2 (x^2-2x)\,dx\right| + \int_2^4 (x^2-2x)\,dx$
$= \left|\left[\frac{x^3}{3} - x^2\right]_0^2\right| + \left[\frac{x^3}{3} - x^2\right]_2^4$
$= \left|\frac{8}{3} - 4\right| + \left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 4\right)$
$= \left|-\frac{4}{3}\right| + \left(\frac{16}{3}\right) - \left(-\frac{4}{3}\right)$
$= 8$ units^2.

7 How many terms are there in the following arithmetic sequence:

$$-12, -10\frac{1}{2}, -9, \ldots, 108$$

A. 67

B. 73

C. 79

(D.) 81

$a = -12$

$d = 1.5$

$T_n = 108$

$a + (n-1)d = 108$

$-12 + 1.5(n-1) = 108$

$-12 + 1.5n - 1.5 = 108$

$1.5n = 121.5$

$n = 81$

8 A and B are events of a sample space.

Given that $P(B|A) = \sqrt{p}$, $P(A) = p$ and $P(B) = p^2$, which of these is an expression for $P(A|B)$?

A. $p^{\frac{3}{2}}$

B. $\sqrt{p}$

C. $\frac{1}{p}$

(D.) $\frac{1}{\sqrt{p}}$

$P(B|A) = \frac{P(B \cap A)}{P(A)}$

$\sqrt{p} = \frac{P(B \cap A)}{p}$

$\therefore P(B \cap A) = p^{3/2}$

$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{p^{3/2}}{p^2}$

$= p^{-1/2}$

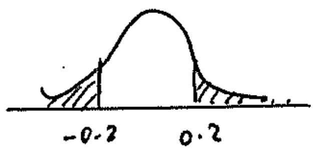
$= \frac{1}{\sqrt{p}}$.

9 A continuous random variable, X, has the following probability density function, where $k > 0$.

$$f(x) = \begin{cases} \sin\left(\frac{\pi}{2} - 2x\right) & \text{for } -k \le x \le k \\ 0 & \text{for all other values} \end{cases}$$

Which of the following is true?

$\sin\left(\frac{\pi}{2} - 2x\right) = \cos 2x$



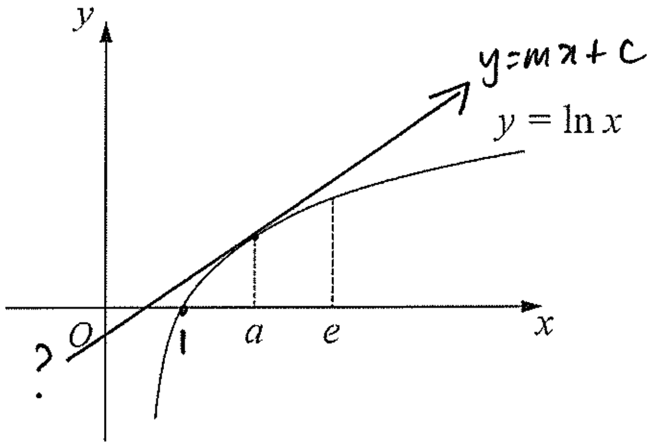
A. $P(X > 0.2) = 1 - P(X \le 0.2)$

(B.) $P(X > 0.2) = P(X < -0.2)$

C. $P(X > 0.2) = P(-0.2 < X < 0.2)$

D. $P(X > 0.2) = P(0 < X < 0.2)$

10 The line $y = mx + c$ is a tangent to the curve $y = lnx$ at the point where $x = a$, as shown in the diagram.



Which of the following statements is true?

A. $\frac{1}{e} < m < 1$ and $-1 < c < 0$

B. $1 < m < e$ and $-1 < c < 0$

C. $\frac{1}{e} < m < 1$ and $0 < c < 1$

D. $1 < m < e$ and $0 < c < 1$

$y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$

at $x = a$: $m = \frac{1}{a}$

at $x = e$: $m = \frac{1}{e}$

at $x = 1$: $m = 1$

$\therefore \frac{1}{e} < m < 1$

(A) or (C)

at $(e, \ln e) \Rightarrow (e, 1)$, $m = \frac{1}{e}$: $y = mx + c$, $1 = \frac{1}{e} \cdot e + c$, $c = 0$.

at $(1, 0)$, $m = 1$: $y = mx + c$, $0 = 1 \times 1 + c$, $c = -1$

$\therefore$ at $(a, \ln a)$ $-1 < c < 0$

(A) ✓

Question 11 (2 marks)

Find integers a and b such that $(3-\sqrt{2})^2 = a + b\sqrt{2}$. 2

$(3-\sqrt{2})^2 = 9 - 6\sqrt{2} + (-\sqrt{2})^2$

$= 9 - 6\sqrt{2} + 2$

$= 11 - 6\sqrt{2}$

$\therefore\ a = 11$ and $b = -6$ (1 mark each)

To obtain the full 2 marks both $a = 11$ and $b = -6$ as the final answer.

Question 12 (2 marks)

Solve the equation $|1-2x| = 5$. 2

$1 - 2x = 5$ or $1 - 2x = -5$

$-2x = 5 - 1$ $-2x = -5 - 1$

$-2x = 4$ $-2x = -6$

$x = -2$ $x = 3$

(1 mark each)

Question 13 (4 marks)

a) Find $\int \frac{x^2}{4x^3-6}dx$ (2)

$$\int \frac{x^2}{4x^3-6}\,dx = \frac{1}{12}\int \frac{12x^2}{4x^3-6}\,dx \checkmark$$

$$= \frac{1}{12}\ln|4x^3-6| + C \checkmark$$

No marks were deducted if absolute value sign was missing.

b) Find $\frac{d}{dx}\left(\frac{\tan x}{e^{2x}}\right)$. Answer in the simplest exact form. (2)

$\frac{d}{dx}\left(\frac{\tan x}{e^{2x}}\right) \frac{u}{v}$ let $u = \tan x$ $\quad v = e^{2x}$

$u' = \sec^2 x \quad v' = 2e^{2x}$

$$\frac{d}{dx}\left(\frac{\tan x}{e^{2x}}\right) = \frac{vu' - uv'}{v^2}$$

$$= \frac{e^{2x}.\sec^2 x - \tan x.\,2e^{2x}}{(e^{2x})^2} \checkmark$$

$$= \frac{e^{2x}(\sec^2 x - 2\tan x)}{(e^{2x})^2}$$

$$= \frac{\sec^2 x - 2\tan x}{e^{2x}} \checkmark$$

To obtain 2 marks
- 1 mark differentiation
- 1 mark simplified.

OR $= \frac{(1+\tan^2 x - 2\tan x)}{e^{2x}}$

$= \frac{(1-\tan x)^2}{e^{2x}}$ OR $\frac{(\tan x - 1)^2}{e^{2x}}$

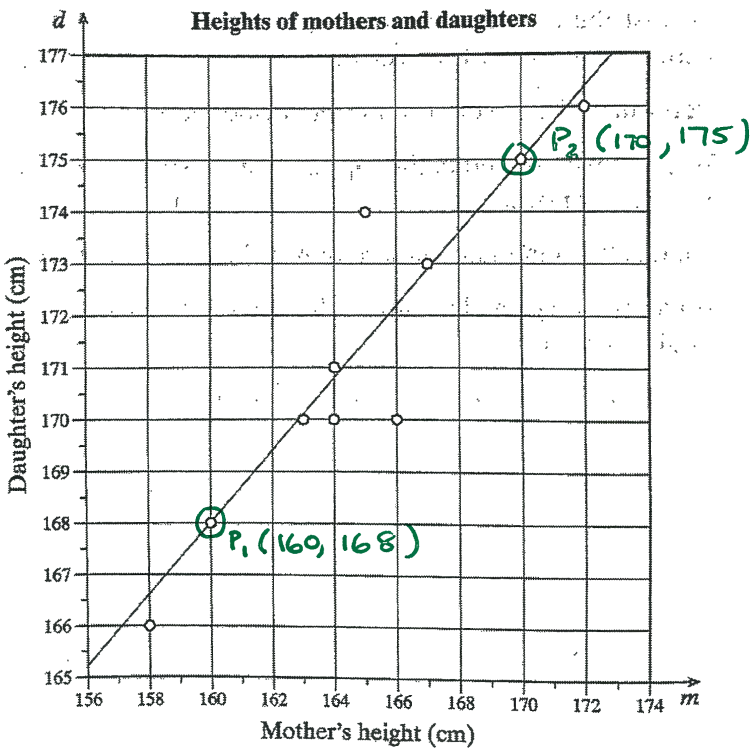
You didn't need to simplify to this.

Question 14 (3 marks)

The heights (cm) of 10 mothers (m) and their daughters (d) were recorded in the table below.

Mother's height in cm (m)	170	163	160	172	164	158	164	166	167	165
Daughter's height in cm (d)	175	170	168	176	170	166	171	170	173	174

The data was used to create a scatterplot and Amelia constructed a regression line by eye, as shown below:



a) Determine the equation of Amelia's regression line. (1)

To obtain the 1 mark equation had to be completely correct

Gradient: $\frac{y_2-y_1}{x_2-x_1} = \frac{175-168}{170-160}$

$m = \frac{7}{10}$

OR Calculator: $y = \frac{7}{10}x + 56$ ✓

Equation of line: $y-y_1 = m(x-x_1)$

$y-168 = \frac{7}{10}(x-160)$

$y-168 = \frac{7}{10}x - 112$

$y = \frac{7}{10}x + 56$

b) Find the value of r, the Pearson's correlation coefficient (correct to 2 decimal places) and hence describe the relationship. (2)

$r = 0.93$ (on calculator) ✓ (1 mark)

strong positive correlation ✓ (1 mark)

both strong & positive had to describe the relationship to obtain the mark.

Question 15 (3 marks)

Evaluate $\int_0^{\frac{\pi}{6}} \left(2\sin x - \sec^2 2x\right) dx$. Leave your answer in the simplest exact form. 3

$$\int_0^{\pi/6} (2\sin x - \sec^2 2x)\,dx = \left[-2\cos x - \tfrac{1}{2}\tan 2x\right]_0^{\pi/6}$$ ✓ (1 mark)

$$= \left[-2\cos(\pi/6) - \tfrac{1}{2}\tan(2\times\tfrac{\pi}{6})\right] - \left[-2\cos 0 - \tfrac{1}{2}\tan 2(0)\right]$$

$$= \left[-2\times\frac{\sqrt{3}}{2} - \frac{1}{2}\times\sqrt{3}\right] - \left[-2(1) - \tfrac{1}{2}(0)\right]$$

$$= -\sqrt{3} - \frac{\sqrt{3}}{2} + 2$$ ✓ (1 mark)

$$= 2 - \frac{3\sqrt{3}}{2} \quad \text{or} \quad \frac{4-3\sqrt{3}}{2}$$ ✓ (1 mark)

(1 mark – integrate
1 mark – substitute / calculate
1 mark – simplify in exact form)

Question 16 (2 marks)

Find the value of $\lim_{x\to 4} \dfrac{x^2-16}{8+2x-x^2}$ 2

$$\lim_{x\to 4} \frac{x^2-16}{8+2x-x^2} = \lim_{x\to 4} \frac{(x-4)(x+4)}{(x-4)(-2-x)}$$

$$= \lim_{x\to 4} \frac{(x-4)(x+4)}{-(x-4)(2+x)}$$

$$= \lim_{x\to 4} \frac{x+4}{-(2+x)}$$ ✓ (1 mark)

$$= \frac{4+4}{-(2+4)}$$

$$= \frac{8}{-6}$$

$$= \frac{4}{-3}$$ ✓ (1 mark)

This was completed poorly as students couldn't factorise the denominator, to then simplify.

Question 17 (6 marks)

Chandra is an enthusiastic gardener. She planted a lemon tree on 1st January 2019, when it was 80 centimetres tall. At the end of the first year after planting, it was 130 centimetres tall, that is, it grew 50 centimetres. Each year's growth was then 90% of the growth of the previous year.

a) How tall was the lemon tree after three years? 2

H, 80, 130, 175,

$H = 80 + (50 + 50 \times 0.9 + 50 \times 0.9^2)$

$= 80 + (50 + 45 + 40.5)$

$= 215.5$ cm

b) Assuming the present growth pattern is maintained, explain why the lemon tree will never reach a height of 10 metres? 2

$a = 50 \quad r = 0.9$

$S_\infty = \dfrac{50}{1 - 0.9}$

$= 500$

max height $= 500 + 80$

$= 580$

$\therefore$ the tree will never reach 10 m

* This question was done poorly many students didn't use the correct series and didn't know, they can't use the 80 as the first term.

c) In which year will the lemon tree reach a height of 5 metres? 2

$$S_n = \frac{a(1-0.9^n)}{1-0.9}$$

$$500 = 80 + \frac{50(1-0.9^n)}{1-0.9}$$

$$420 = \frac{50(1-0.9^n)}{0.1}$$

$$0.84 = 1 - 0.9^n$$

$$0.9^n = 0.16$$

$$n \ln 0.9 = \ln 0.16$$

$$n = \frac{\log 0.16}{\log 0.9}$$

$$n = 17.3$$

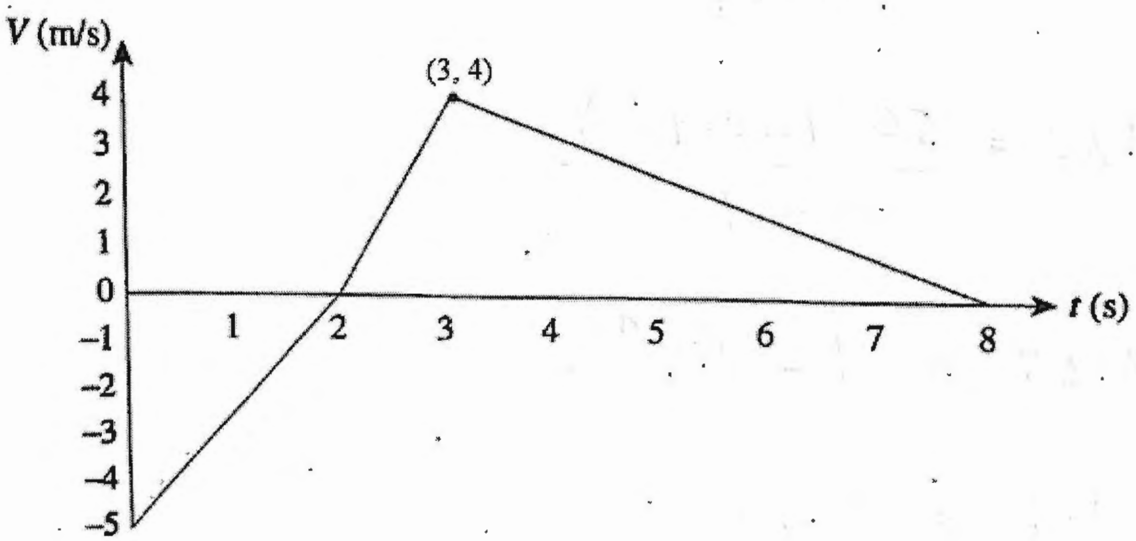
during 18^{th} year

$\therefore$ 2036

* This question also was done poorly some students didn't even use the sum formula properly.

Question 18 (3 marks)

The graph below is composed of three line segments and shows the velocity V of a particle which is moving in a straight line. The velocity is given in metres per second at time t seconds, where $0 \le t \le 8$.



a) Determine the total distance covered by the particle during the 8 seconds. 2

$d = \frac{1}{2} \times 2 \times 5 + \frac{1}{2}(4 \times 6)$

$= 5 + 12$

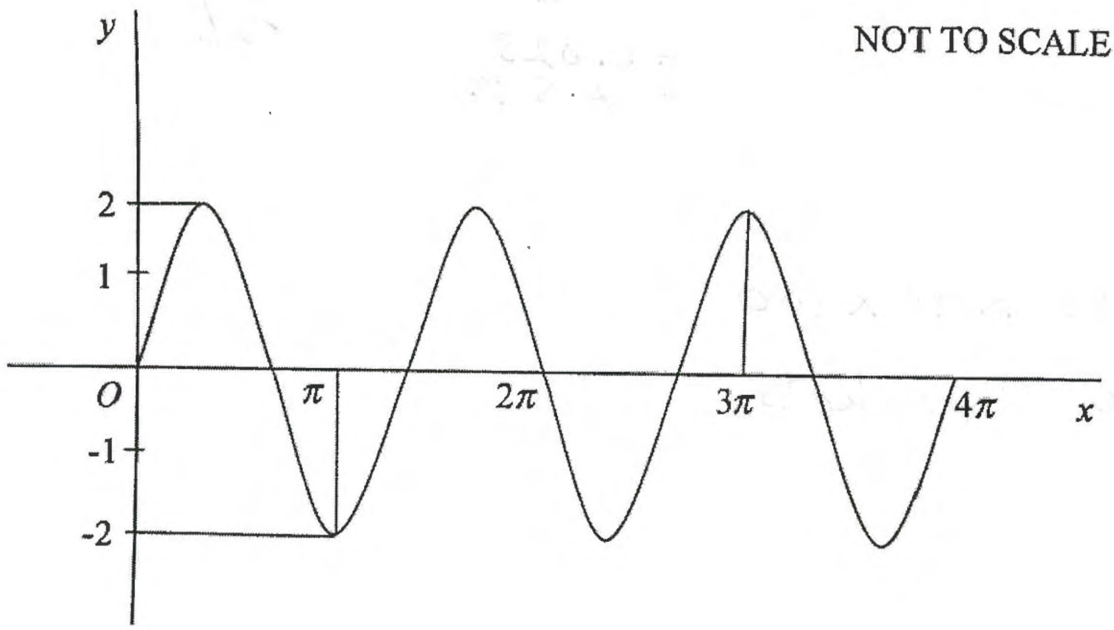
$= 17$ m

b) What is the particle's position relative to its starting position when $t = 8$ seconds? 1

7 cm to the right

Question 19 (2 marks)

Find the equation for the graph given below: 2



$a = 2$

$\frac{4\pi}{3} = \frac{2\pi}{b}$ $y = 2 \sin \frac{3}{2} x$

$4b = 6$

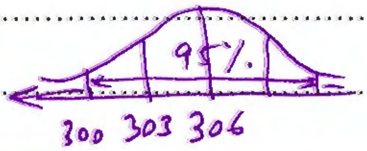
$b = \frac{3}{2}$

many students didn't know how to find b.

Question 20 (4 marks)

Packets of coffee beans are labelled with a net weight of 300 g. It is found that the weight of a packet can be modelled by a normal distribution, with mean 306 g and standard deviation 3 g.

a) Use the Empirical rule to determine the probability that the weight of one packet of coffee beans is less than the advertised weight of 300 g. 1



(P < 2σ below the mean) = $\frac{0.05}{2}$

= 0.025

= 2.5%

* Some students didn't use Empirical rule

b) In a shipment of 40 boxes, each with 100 packets of coffee beans, how many packets would be expected to be underweight? 1

= 0.025 × 40 × 100

= 100 packets

c) Manufacturers aim to ensure that the expected number of underweight packets in the shipment will be less than 20. The machine is adjusted to give a mean weight of 308 g, with the standard deviation of 3 g remaining the same. Will they meet their target? Justify your answer. 2

$Z = \frac{300 - 308}{3}$

$= -2.\dot{6}$ $P(z < -2.\dot{6})$

$P(z < -2.\dot{6}) = 1 - P(z < 2.6)$

$= 1 - 0.996$

$= 0.004$

$= 0.4\%$

= 0.004 × 4000

= 16

They meet the target 16 < 20

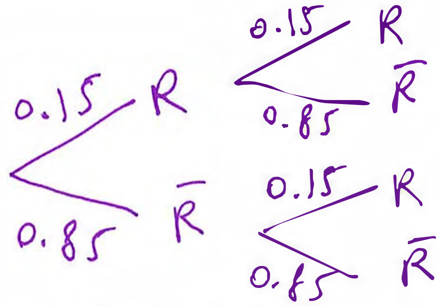
Some students had difficulty with this question

Question 21 (4 marks)

The probability it will rain on any day in June is 0.15. Two days in June are selected at random.

a) Complete the probability distribution table, where the variable X represents the number of rainy days across the 2 days: 1

x	0	1	2
$P(X = x)$	0.7225	0.255	0.0225



* many students didn't know how to complete the table you had to use a tree diagram.

b) Calculate, correct to two decimal places:

i) the expected value.

$\mu = 1 \times 0.255 + 2 \times 0.0225$

$= 0.30$

ii) the variance and standard deviation. 2

$Var = (1^2 \times 0.255 + 2^2 \times 0.0225) - (0.30)^2$

$= 0.255$

$= 0.26$

$\sigma = 0.5099$

$= 0.51$

Question 22 (4 marks)

Solve the equation $\sec\theta + 8\cos\theta = 6$ for $0 \le \theta \le 2\pi$. Answer correct to 3 decimal places, where necessary. 4

$\sec Q + 8\cos Q = 6$

$\frac{1}{\cos Q} + 8\cos Q = 6$

$8\cos^2 Q - 6\cos Q + 1 = 0$ ✓

Let $x = \cos Q$

$8x^2 - 6x + 1 = 0$

$8x^2 - 4x - 2x + 1 = 0$

$4x(2x-1) - 1(2x-1) = 0$

$(4x-1)(2x-1) = 0$

$x = \frac{1}{4} \quad x = \frac{1}{2}$

✓ ✓ $\therefore \cos Q = \frac{1}{4} \qquad \cos Q = \frac{1}{2}$ ✓

$Q = 1.318, 4.965, \frac{\pi}{3}, \frac{5\pi}{3}$

$= \frac{\pi}{3}, 1.318, 4.965, \frac{5\pi}{3}$ ✓✓

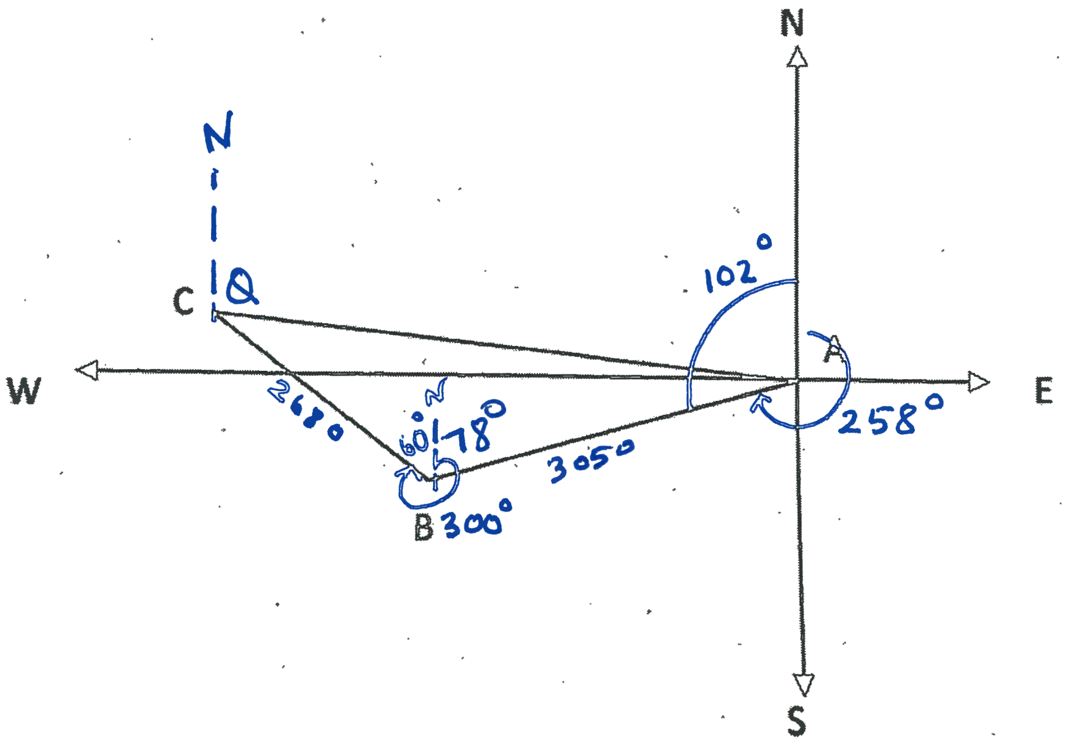
Solutions needed to be in radians for full marks.

Question 23 (4 marks)

A plane takes off from an airport (A) and travels in a direction of 258° for 3050 kilometres.

The plane lands at (B) and then heads in a direction of 300° for 2680 kilometres, landing at (C).

Use the diagram below to mark the given information.



a) Find the distance the plane must travel to return to A. Answer to the nearest kilometre. 2

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 2680^2 + 3050^2 - 2(2680)(3050)\cdot\cos 138°$ ✓

$b \doteqdot 5351$ km (nearest km) ✓

Common problems with this question involved not knowing the cosine rule and incorrectly calculating ∠CBA.

b) Find the bearing that the plane must travel from C to A. 2

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{5351^2 + 3050^2 - 2680^2}{2 \times 5351 \times 3050}$

$A \doteqdot 19^\circ 35'$ (nearest min)

$\alpha = 102 - 19^\circ 35'$
$\doteqdot 82^\circ 25'$

$\theta = 180 - 82^\circ 25'$
$\doteqdot 97^\circ 35'$

$\therefore$ Bearing is approx $098^\circ T$ (nearest degree)

Alternative solution:

$\cos C = \dfrac{2680^2 + 5351^2 - 3050^2}{2 \times 2680 \times 5351}$

$C \doteqdot 22^\circ 25'$

$\therefore$ Bearing $= 120 - 22^\circ 25'$
$\doteqdot 97^\circ 35'$

Question 24 (3 marks)

The table below shows the future values of an annuity of $1 for different rates of interest and for different numbers of compounding periods. The contributions are made at the end of each compounding period.

Future Value Interest Factors

Time Period	*Interest Rate*				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

An annuity account is opened and contributions of $500 are made at the end of every six months for 5 years.

For the first 4 years, the interest rate is 6% per annum, compounding six-monthly. For the 5th year, the interest rate increases to 8% per annum, compounding six-monthly.

Calculate the amount in the account immediately after the last contribution is made. 3

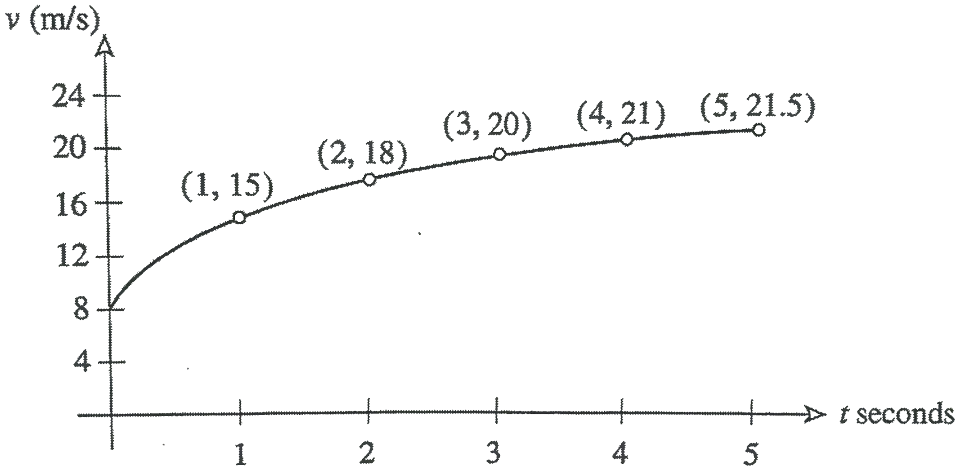
Total after 4 years $= 500 \times 8.8923$
$= \$4446.15$ ✓

Total after 5 years $= 4446.15(1.04)^2 + 500(2.04)$ ✓ ✓
$= \$5828.96$
[or $= 4446.15(1.04)^2 + 500(1.04) + 500$.]

Students had trouble with this question and many did not know how to use the table. A common mistake was forgetting to multiply $4446.15 \times (1.04)^2$ to include interest for the final year.

Question 25 (4 marks)

The diagram shows the graph of a particle's velocity, v m/s, at time t seconds.



a) Use the trapezoidal rule with 3 sub-intervals (4 function values) to approximate the distance the particle travels in the first 3 seconds. 2

$D = \frac{h}{2}\left[f(1st) + f(last) + 2f(middle)\right]$

$= \frac{1}{2}\left[8 + 20 + 2(15+18)\right]$

$= 47\text{m}$

* Some students do not know how to apply the trapezoidal rule.

b) Is the estimate for your answer in part a) more than or less than the exact distance that the particle travels in the first 3 seconds? Justify your answer. 2

The estimate is less than the exact distance since the velocity time graph is concave down and the trapezium create chords that lie underneath the graph. Hence the estimate does not take into account the area between the trapeziums and the graph.

Needed to discuss area for full marks.

Question 26 (4 marks)

Consider the cubic function $y = x^3 + ax^2 + bx + 3$, where a and b are integers.

At the point $(1, 8)$ on the curve, the equation of the tangent is given by $y = 2x + 6$.

Determine the values of a and b. 4

Let $(x, y) = (1, 8)$

$8 = 1 + a + b + 3$

$a + b = 4$ ①

when $x = 1$ $y' = 2$

$y' = 3x^2 + 2ax + b$

$2 = 3 + 2a + b$

$2a + b = -1$ ②

② − ①

$a = -5$ sub in ①

$-5 + b = 4$

$b = 9$

Question 27 (6 marks)

A probability density function is given by $f(x) \begin{cases} \dfrac{a}{\sqrt{1+4x}}; & 0 \le x \le 2 \\ 0 & ; \text{otherwise} \end{cases}$ where a is a constant.

a) Show that $a = 1$. 2

$$\int_0^2 \frac{a}{\sqrt{1+4x}} = 1 \quad ✓$$

$$\int_0^2 a(1+4x)^{-1/2} = 1$$

$$\left[\frac{a(1+4x)^{1/2}}{\frac{1}{2}\times 4}\right]_0^2 = 1$$

$$\frac{a}{2}\sqrt{9} - \frac{a}{2}\sqrt{1} = 1 \quad ✓$$

$$\frac{3a}{2} - \frac{a}{2} = 1$$

$$\frac{2a}{2} = 1$$

$$a = 1$$

b) Find the cumulative distribution function, $F(x)$. 2

CDF: $F(x) = \int_0^x (1+4t)^{-1/2}\,dt$ ✓

$$= \left[\frac{(1+4t)^{1/2}}{\frac{1}{2}\times 4}\right]_0^x$$

$$\therefore F(x) = \frac{\sqrt{1+4x}}{2} - \frac{1}{2} \quad ✓$$

$$= \frac{1}{2}\left(\sqrt{1+4x} - 1\right)$$

* Students need to set-up the first step properly using the second variable and limits.

* Note the complete CDF is:

$$F(x) = \begin{cases} 1, & x > 2 \\ \frac{1}{2}\left(\sqrt{1+4x} - 1\right), & 0 \le x \le 2 \\ 0, & x < 0. \end{cases}$$

c) Find the median. 2

Median: $CDF = \frac{1}{2}$

$\frac{1}{2}\left(\sqrt{1+4x} - 1\right) = \frac{1}{2}$ ✓

$\sqrt{1+4x} - 1 = 1$

$\sqrt{1+4x} = 2$

$1 + 4x = 4$

$4x = 3$

$x = \frac{3}{4}$ ✓

Question 28 (3 marks)

The rate of 'flu infection' in a population of a city is proportional to the number of infected individuals. That is the number of infected people F after t weeks is modelled by the equation $F = F_0 e^{kt}$ where F_0 and k are constants. It is known that after 3 weeks, there is twice the number of infections to begin with. If there were 1000 cases of flu infection originally, how many are there after 7 weeks? Express your answer correct to three significant figures. 3

$F = F_0 e^{kt}$

when $t=3$; $F = 2F_0$; $F_0 = 1000$

$2000 = 1000e^{3k}$

$2 = e^{3k}$

$\ln 2 = 3k \ln e$

$k = \frac{\ln 2}{3}$ ✓

$F = 1000e^{\frac{\ln 2}{3}t}$

when $t = 7$:

$F = 1000e^{\frac{7\ln 2}{3}}$ ✓

$= 5039.6842$

$\doteqdot 5000$ (3 sig. figs).

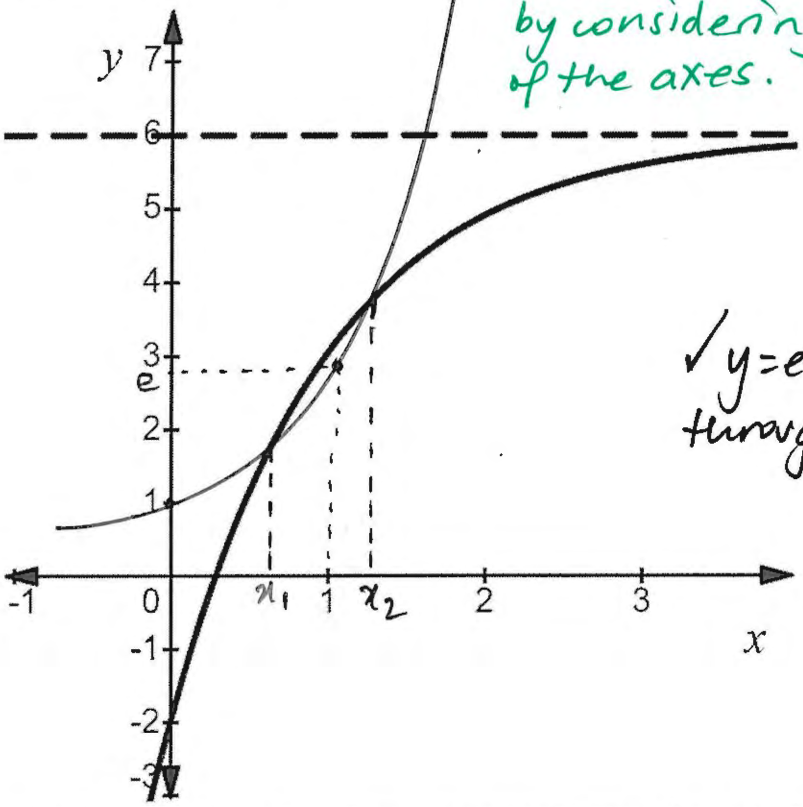
$\therefore$ 5000 cases of flu ✓

*Rounding correctly was necessary for this question.

Question 29 (6 marks)

The diagram shows the graph of $y = 6 - 8e^{-x}$.

Students could improve their sketch of $y=e^x$ by considering the SCALE of the axes.



✓ $y=e^x$ passes through $(1,e)$ and $(0,1)$.

a) On the diagram, sketch the graph of $y = e^x$, which passes through the point $(1, e)$. Show that the x coordinates of the two points of intersection, (x_1 and x_2, where $x_1 < x_2$), are the solutions of the equation $e^{2x} - 6e^x + 8 = 0$ and solve this equation to find the exact values of x_1 and x_2. 4

Curves intersect at:

$$e^x = 6 - 8e^{-x}$$

$$e^x = 6 - \frac{8}{e^x}$$

$$e^x = \frac{6e^x - 8}{e^x} \checkmark$$

$$e^{2x} = 6e^x - 8$$

$$\therefore e^{2x} - 6e^x + 8 = 0$$

Many students did not show this step

$$e^{2x} - 6e^x + 8 = 0$$

$$(e^x - 2)(e^x - 4) = 0$$

$$e^x = 2 \quad \text{or} \quad e^x = 4$$

$$x \ln e = \ln 2 \qquad x \ln e = \ln 4$$

$$x_1 = \ln 2 \checkmark \qquad x_2 = \ln 4 \checkmark$$

b) Find in simplest exact form, the area of the region enclosed by the two curves. 2

$$\text{Area} = \int_{\ln 2}^{\ln 4} (6 - 8e^{-x} - e^{x})\,dx$$

$$= \left[6x - \frac{8e^{-x}}{-1} - e^{x}\right]_{\ln 2}^{\ln 4}$$

$$= \left[6x + 8e^{-x} - e^{x}\right]_{\ln 2}^{\ln 4} \checkmark$$

$$= 6(\ln 4 - \ln 2) + 8(e^{-\ln 4} - e^{-\ln 2}) - (e^{\ln 4} - e^{\ln 2})$$

$$= 6\left(\ln\left(\tfrac{4}{2}\right)\right) + 8(e^{\ln 1/4} - e^{\ln 1/2}) - (e^{\ln 4} - e^{\ln 2})$$

$$= 6\ln 2 + 8\left(\tfrac{1}{4} - \tfrac{1}{2}\right) - (4 - 2)$$

$$= 6\ln 2 + 8\left(-\tfrac{1}{4}\right) - 2$$

$$= 6\ln 2 - 2 - 2$$

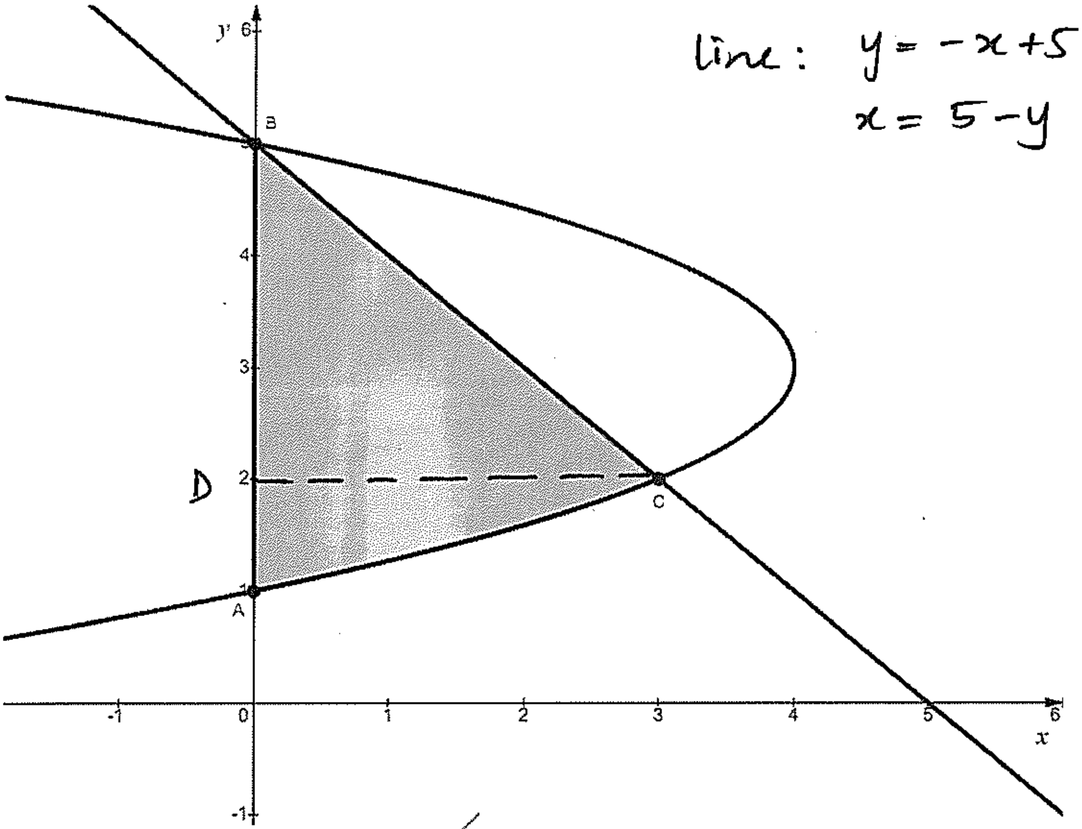
$$= 6\ln 2 - 4$$

$$\therefore \text{Area} = (6\ln 2 - 4) \text{ units}^2 \checkmark$$

* Students should always calculate their final answer to see if it is POSITIVE for finding area!!

Question 30 (3 marks)

In the diagram below, the parabola $x = -y^2 + 6y - 5$ meets the y –axis at points $A(0,1)$ and $B(0,5)$. The line $y = -x + 5$ meets the parabola at points B and $C(3,2)$. Find the shaded area, which is bounded by the parabola, the line and the y –axis. Leave your answer as a simplified fraction. 3



line: $y = -x + 5$

$x = 5 - y$

$$\text{Area} = \int_2^5 (5-y)\,dy + \int_1^2 (-y^2 + 6y - 5)\,dy$$

$$= \left[5y - \frac{y^2}{2}\right]_2^5 + \left[\frac{-y^3}{3} + \frac{6y^2}{2} - 5y\right]_1^2$$

$$= \left(25 - \frac{25}{2}\right) - \left(10 - \frac{4}{2}\right) + \left(-\frac{8}{3} + 3\times4 - 5\times2\right) - \left(-\frac{1}{3} + 3 - 5\right)$$

$$= \frac{25}{2} - 8 + \left(-\frac{2}{3}\right) - \left(-\frac{7}{3}\right)$$

$$= 4\frac{1}{2} + \frac{5}{3}$$

$$= \frac{37}{6}$$

$\therefore$ Area $= 6\frac{1}{6}$ units^2.

Alternate Solution:

$$Area = Area\ \Delta BCD + \int_1^2 (-y^2+6y-5)\,dy$$

$$= \frac{1}{2}\times BD\times CD + \left[\frac{-y^3}{3}+\frac{6y^2}{2}-5y\right]_1^2$$ ✓

$$= \frac{1}{2}\times 3\times 3 + \left(-\frac{8}{3}+3\times 4-10\right)-\left(-\frac{1}{3}+3-5\right)$$ ✓

$$= \frac{9}{2} + \left(-\frac{2}{3}\right) - \left(-\frac{7}{3}\right)$$

$$= \frac{9}{2}+\frac{5}{3}$$

$$= \frac{37}{6}$$ ✓

Area $= 6\frac{1}{6}$ units^2

* Other solutions were accepted.

* This question was marked generously.

Question 31 (5 marks)

Karen is retiring next week and her Superannuation Fund contains $1 200 000. The Fund is earning 6% p.a. compound interest, compounding monthly. To cover her living expenses in her retirement, Karen wishes to withdraw a regular amount of $8 000 at the end of each month, after interest has been added.

a) Show that after 3 months the amount in her account A_3 is given by:

$A_3 = 1\,200\,000(1.005)^3 - 8\,000[(1.005)^2 + (1.005) + 1]$ 2

$A_1 = 1200000(1.005) - 8000$

$A_2 = A_1(1.005) - 8000$

$= [1200000(1.005) - 8000](1.005) - 8000$

$= 1200000(1.005)^2 - 8000(1.005) - 8000$ ✓

$A_3 = A_2(1.005) - 8000$

$= [1200000(1.005)^2 - 8000(1.005) - 8000](1.005) - 8000$ ✓

$= 1200000(1.005)^3 - 8000(1.005)^2 - 8000(1.005) - 8000$

$\therefore A_3 = 1200000(1.005)^3 - 8000[(1.005)^2 + (1.005) + 1]$, as required.

* The expanding process needs to be shown.

b) By finding a similar expression for the amount remaining after n months, find how many years the money will last. 3

Similarly:

$$A_n = 1200\,000(1.005)^n - 8000\left[(1.005)^{n-1} + \dots + (1.005) + 1\right]$$

G.P. sum: $a = 1$, $r = 1.005$, $n = n$ terms

✓ expression in 'n'.

The money will last until $A_n = 0$. Find n:

$$0 = 1200\,000(1.005)^n - \frac{8000 \times 1 \times (1.005^n - 1)}{1.005 - 1}$$

✓ correct G.P. sum formula.

$$\therefore 1200\,000(1.005)^n = \frac{8000(1.005^n - 1)}{0.005}$$

$$1200(1.005)^n = 1600(1.005^n - 1)$$

$$1200(1.005)^n = 1600(1.005)^n - 1600$$

$$400(1.005)^n = 1600$$

$$(1.005)^n = 4$$

$$n \ln(1.005) = \ln 4$$

$$n = \frac{\ln 4}{\ln(1.005)}$$

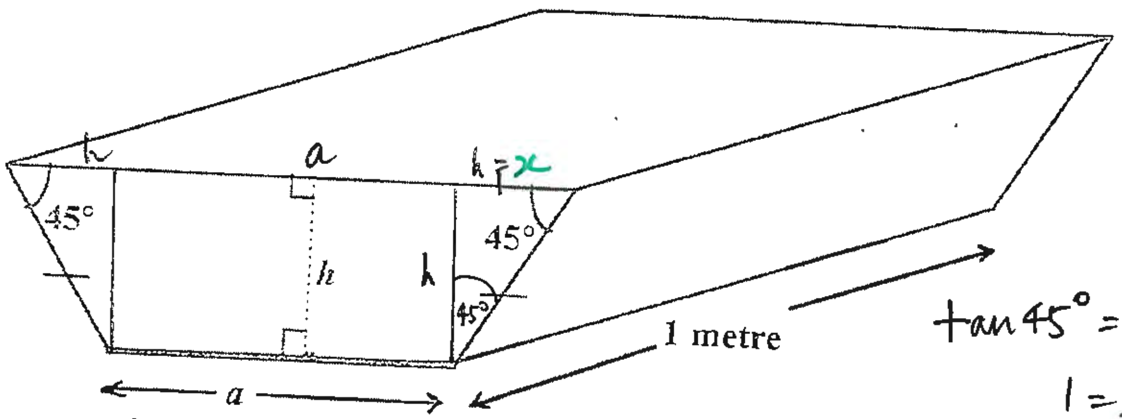
✓ $n = 277.95$ months

$n \doteqdot 23$ years (nearest year)

either answer without error.

* Students should ensure that their final answer is a good estimate with regards to life expectancy!

Question 32 (6 marks)



$\tan 45° = \frac{h}{x}$

$1 = \frac{h}{x}$

$\therefore x = h$.

(DIAGRAM NOT DRAWN TO SCALE)

$\therefore b = a + 2x$

An open trough of depth h metres and length one metre is constructed out of stainless steel sheeting.

The cross-section of the trough is an isosceles trapezium with the acute angles being 45° each. $b = a + 2h$

The width of the bottom of the trough is a metres. The area of the cross-section measures 60 m^2.

a) Show that $a = \dfrac{60}{h} - h$. 2

Area of crosssection $= \frac{1}{2} \times h \times (a + b)$

$60 = \frac{1}{2} \times h \times (a + a + 2h)$ ✓

$60 = \frac{1}{2} \times h \times (2a + 2h)$

$120 = 2ah + 2h^2$

$2ah = 120 - 2h^2$

$a = \dfrac{120}{2h} - \dfrac{2h^2}{2h}$ ✓

$\therefore a = \dfrac{60}{h} - h$, as required.

b) Show that the amount of stainless steel, A, in m^2, required to construct the trough is given by:

$A = \frac{60}{h} - h + 2h\sqrt{2} + 120$. 2

$2 \times 60 \leftarrow$ from question.

$A = 2 \times \left(\frac{1}{2} \times h \times (2a + 2h)\right) + 2 \times \left(\sqrt{h^2 + h^2} \times 1\right) + (a \times 1)$ ✓

$= h(2a + 2h) + 2\sqrt{2h^2} + a$

$= 2h\left(\frac{60}{h} - h\right) + 2h^2 + 2\sqrt{2}\,h + \left(\frac{60}{h} - h\right)$

$= (120) - 2h^2 + 2h^2 + 2h\sqrt{2} + \left(\frac{60}{h} - h\right)$ ✓

$\therefore A = \frac{60}{h} - h + 2h\sqrt{2} + 120$, as required.

* Other correct approaches were accepted.

c) Find the depth of the trough, to the nearest millimetre (mm), if the amount of stainless steel used is kept to a minimum. 2

$A = 60h^{-1} - h + 2h\sqrt{2} + 120$

$\frac{dA}{dh} = -60h^{-2} - 1 + 2\sqrt{2}$

$\frac{d^2A}{dh^2} = 120h^{-3} = \frac{120}{h^3}$

For minimum height $\frac{dA}{dh} = 0$: Find h

$0 = -\frac{60}{h^2} - 1 + 2\sqrt{2}$

$\frac{60}{h^2} = -(1 - 2\sqrt{2})$

$h^2 = \frac{-60}{1-2\sqrt{2}}$

$h = \sqrt{\frac{60}{2\sqrt{2}-1}} \doteqdot \sqrt{32.815..} \doteqdot 5.728$ m (nearest mm) ✓

$\frac{d^2A}{dh^2} = \frac{120}{(5.728)^3} = 0.638 > 0$ ∪ ∴ minimum.

∴ $h \doteqdot 5.728$ m ✓, for the stainless steel used to be kept to a minimum.

* ① Mark awarded for the correct process, as too many errors were made.

* Students should ensure that their answers are relative to the measurement scale in the question.

Question 33 (5 marks)

A prototype rocket which is initially at rest, takes off from a launchpad on the ground.

It has a time of flight of T seconds, and t is the time in seconds, where $0 \leq t \leq T$.

The velocity of the rocket, v ms^{-1}, is given by:

$$v(t) = 0.5e^t \sin\left(\frac{\pi t}{10}\right)$$

a) Shortly after the rocket takes off, the engine stops and it begins to descend towards the ground.

Find the time at which the rocket begins to descend. 2

$V = 0$ when the engine stops. Find t:

$0 = 0.5e^t \sin\left(\frac{\pi t}{10}\right)$ ✓

$0.5e^t \neq 0$ no solution ← This should be noted.

$\therefore \sin\left(\frac{\pi t}{10}\right) = 0$

$\frac{\pi t}{10} = 0, \pi, 2\pi, \ldots$

$\frac{t}{10} = 0, 1, 2, \ldots$ ✓

$\therefore t = 10$ seconds, when the rocket begins to descend towards the ground.

* Many students did not understand part a) and commenced with the answer for part b).

b) Before the rocket starts to descend it reaches its maximum velocity. Find the time it takes for the rocket to achieve its maximum velocity. Give your answer correct to two decimal places. 3

$\frac{dv}{dt} = 0$ for maximum velocity.

$v(t) = 0.5e^t \sin\left(\frac{\pi t}{10}\right)$

$\frac{dv}{dt} = 0.5e^t \times \frac{\pi}{10}\cos\left(\frac{\pi t}{10}\right) + \sin\left(\frac{\pi t}{10}\right) \times 0.5e^t$

$0 = 0.5e^t\left(\frac{\pi}{10}\cos\left(\frac{\pi t}{10}\right) + \sin\left(\frac{\pi t}{10}\right)\right)$ ✓

$0.5e^t \neq 0$ no solution

$\therefore \frac{\pi}{10}\cos\left(\frac{\pi t}{10}\right) + \sin\left(\frac{\pi t}{10}\right) = 0$

$\sin\left(\frac{\pi t}{10}\right) = -\frac{\pi}{10}\cos\left(\frac{\pi t}{10}\right)$

$\therefore \tan\left(\frac{\pi t}{10}\right) = -\frac{\pi}{10}$ ✓

$\frac{\pi t}{10} = -0.304\ldots, 2.837\ldots,$ etc

$\therefore t = 2.837\ldots \times \frac{10}{\pi}$

$t \doteq 9.03$ seconds (2dp) ✓

$\therefore$ rocket reaches maximum velocity at 9.03 seconds.

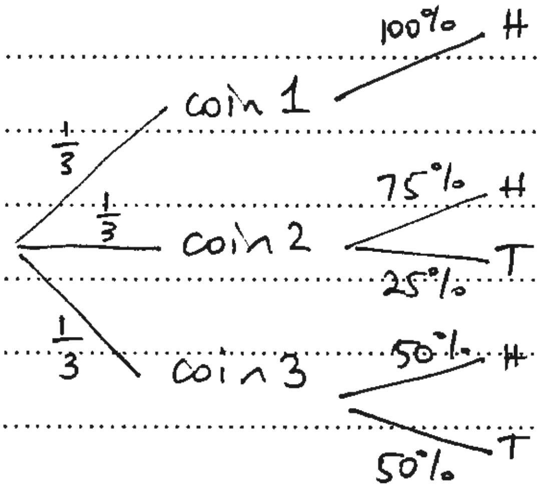
* Students did not attempt this question well. No carry-on errors were accepted, marks awarded as indicated.

Question 34 (2 marks)

The Artful Dodger has three 20 cent coins in his pocket. One of the coins has heads on both sides, another is biased such that it has a 75% chance of landing on heads and the third coin is a fair coin.

A coin is selected at random and tossed.

Given that the coin that was tossed comes up heads, what is the probability that it was the fair coin? 2



$$P(\text{fair} \mid \text{heads})$$

$$= \frac{P(\text{fair} \cap \text{heads})}{P(\text{heads})}$$

$$= \frac{\frac{1}{3} \times \frac{1}{2}}{\left(\frac{1}{3} \times 1\right) + \left(\frac{1}{3} \times \frac{3}{4}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right)}$$

✓ either numerator or denominator.

$$= \frac{\frac{1}{6}}{\frac{1}{3} + \frac{1}{4} + \frac{1}{6}}$$

$$= \frac{2}{4 + 3 + 2}$$

$$= \frac{2}{9}$$

✓ correct answer

The End.